Eur. Phys. J. B **50**, 285–289 (2006)
DOI: 10.1140/epjb/e2006-00056-6

THE EUROPEAN PHYSICAL JOURNAL B

Gravitational structure formation, the cosmological problem and statistical physics

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Received 11 October 2005 / Received in final form 18 November 2005
Published online 16 February 2006 –

Abstract. Models of structure formation in the universe postulate that matter distributions observed today in galaxy catalogs arise, through a complex non-linear dynamics, by gravitational evolution from a very uniform initial state. Dark matter plays the central role of providing the primordial density seeds which will govern the dynamics of structure formation. We critically examine the role of cosmological dark matter by considering three different and related issues: Basic statistical properties of theoretical initial density fields, several elements of the gravitational many-body dynamics and key correlation features of the observed galaxy distributions are discussed, stressing some useful analogies with known systems in modern statistical physics.

PACS. 98.80.-k Cosmology – 05.70.-a Thermodynamics – 02.50.-r Probability theory, stochastic processes, and statistics – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion

1 Introduction

The large distribution of matter in the universe as traced by galaxy structures shows a complex irregular pattern, characterized by clusters of galaxies which are organized in filaments around large voids. In the framework of standard cosmological models these structures arise from non-linear dynamical evolutions in which Newtonian gravity plays the essential role (e.g., [1]). The topic of structure formation in the expanding universe is clearly extremely large and we focus here on three different and related issues, where the methods and concepts of statistical physics find a fruitful applications [2].

2 Primordial density fields

The first topic concerns the statistical properties of initial matter density fields in standard cosmological models: as we discuss below, in the framework of Friedmann-Robertson-Walker (FRW) [1] models of cosmological expansion, there are important constraints which have to be satisfied by primordial matter fluctuation fields and which are common to all models and independent on the nature of dark matter [3]. Some very interesting analogies with glassy systems (e.g. one component plasma) will be outlined and the crucial observational tests of standard models of cosmological structure formation will be discussed.

Dark matter plays the major role in the problem of structure formation in standard cosmological models: large-scale structures we observe today must have formed from the effects of gravity acting on small amplitude seed fluctuations in the original distribution of dark matter from the Big Bang. The problem of structure formation in the universe is approximately Newtonian but embedded in an expanding universe, whose dynamics is described by General Relativity. The role and amount of dark matter is determined to make compatible different types of astronomical observations with the FRW models. Without entering into the details of the various reasons why dark matter is fundamental in the cosmological context it is worth noticing here that the most "popular" model, which nowadays is the so-called Λ-Cold Dark Matter (or ΛCDM), postulates the structure of mass and energy in a Universe as 5% ordinary baryonic matter, 25% CDM of non-baryonic form (which has not been yet detected in laboratories on Earth), and 70% dark energy, which would be a uniform component of energy repelling to gravity and pushing the Universe apart faster. While this latter energy component is essentially relevant only for the rate of expansion in FRW models, the CDM would be important for structure formation and ordinary baryonic matter does

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not make any relevant dynamical effect and it just follows the distribution of CDM.

Primordial density fluctuations have imprinted themselves on the patterns of radiation also, and those variations should be detectable in the Cosmic Microwave Background Radiation (CMBR). Three decades of observations have revealed fluctuations in the CMBR of amplitude of order 10^{-5} [4]. It is in fact to make these measurements compatible with observed structures that it is necessary to introduce non-baryonic dark matter which interact with photons only gravitationally, and thus in a much weaker manner than ordinary baryonic matter. Thus in these models dark matter plays the dominant role of providing density fluctuation seeds which, from the one hand are compatible with observations of the CMBR and from the other hand they are large enough to allow the formation, through a complex non linear dynamics, of galaxy structures we observe today.

A simple calculation may give the order of magnitude of the initial amplitude of fluctuations. In the linear regime small amplitude fluctuations grow inversely proportional to the redshift z in the expanding universe. Thus given that the CMBR is at $z = 10^3$ (i.e. early cosmological times), in order to have density fluctuations $\delta\rho/\rho$ of order one today (i.e. $z = 0$) on a certain scale (and larger than the average on smaller scales to make non-linear structures) one should have $\delta\rho/\rho \approx 10^{-3}$ at $z = 10^3$ at the corresponding scale. However there is a factor 10^2 of difference given that $\delta T/T \approx 10^{-5}$ and $\delta\rho/\rho \approx \delta T/T$ for ordinary baryonic matter. As mentioned, CDM interacts weakly with radiation making possible to have $\delta\rho/\rho \approx 10^{-3}$ at $z = 10^3$ and in the same time $\delta T/T \approx 10^{-5}$. This would not be possible with ordinary baryonic matter. Therefore properties of dark matter in this cosmological context are defined to allow structure formation today.

From the above discussion it seems that much freedom is left for the choice of dark matter, its physical properties and its statistical distribution, unless it will once be directly observed. However there is an important constraint which must be valid for any kind of initial matter density fluctuation field in the framework of FRW models. This must be imprinted in the CMBR, a relict of the high energy process occurred in the early universe according to standard models.

The most prominent feature of theoretical models of the initial conditions derived from inflationary mechanisms is that matter density field presents on large scale super-homogeneous features [3]. This means the following. If one considers the paradigm of uniform distributions, the Poisson process where particles are placed completely randomly in space, the mass fluctuations in a sphere of radius R grows as R^3, i.e. like the volume of the sphere. A super-homogeneous distribution is a system where the average density is well defined (i.e. it is uniform) and where fluctuations in a sphere grow slower than in the Poisson case, e.g. like R^2: in this case there are the so-called surface fluctuations to differentiate them from Poisson-like volume fluctuations. (Note that a uniform system with positive correlations present fluctuations which grow faster than Poisson.) For example a perfect cubic lattice of particle is a super-homogeneous system. An example of a well known system in statistical physics systems of this kind is the one component plasma [5] which is characterized by a dynamics which at thermal equilibrium gives rise to such configurations. In the cosmological context inflationary models predict a spectrum of fluctuations of this type.

The reason for this peculiar behavior of primordial density fluctuations is the following. In a FRW cosmology there is a fundamental characteristic length scale, the horizon scale $R_H(t)$. It is simply the distance light can travel from the Big Bang singularity $t = 0$ until any given time t in the evolution of the Universe, and it grows linearly with time. The Harrison-Zeldovich (H-Z) criterion states that the normalized mass variance at the horizon scale is constant: this can be expressed more conveniently in terms of the power spectrum (PS) of density fluctuations [3] $P(\boldsymbol{k}) = \left\langle |\delta_\rho(\boldsymbol{k})|^2 \right\rangle$ where $\delta_\rho(\boldsymbol{k})$ is the Fourier Transform of the normalized fluctuation field $(\rho(\boldsymbol{r}) - \rho_0)/\rho_0$, being ρ_0 the average density. It is possible to show that the H-Z-criterion is equivalent to assume $P(k) \sim k$: in this situation matter distribution present surface fluctuations [3].

In order to illustrate more clearly the physical implications of this condition, one may consider gravitational potential fluctuations $\delta\phi(\boldsymbol{r})$ which are linked to the density fluctuations $\delta\rho(\boldsymbol{r})$ via the gravitational Poisson equation: $\nabla^2 \delta\phi(\boldsymbol{r}) = -4\pi G \delta\rho(\boldsymbol{r})$. From this, transformed to Fourier space, it follows that the PS of the potential $P_\phi(k) = \left\langle |\delta\hat{\phi}(\boldsymbol{k})|^2 \right\rangle$ is related to the density PS $P(k)$ as: $P_\phi(k) \sim \frac{P(k)}{k^4}$ The H-Z condition corresponds therefore to $P_\phi(k) \propto k^{-3}$ so that gravitational potential fluctuations become constant as a function of scale.

The H-Z condition is a consistency constraint in the framework of FRW cosmology. In fact the FRW is a cosmological solution for a homogeneous Universe, about which fluctuations represent an inhomogeneous perturbation: if density fluctuations obey to a different condition than the H-Z criterion, then the FRW description will always break down in the past or future, as the amplitude of the perturbations become arbitrarily large or small. For this reason the super-homogeneous nature of primordial density field is a fundamental property independently on the nature of dark matter. We note that this is a very strong condition to impose, and it excludes even Poisson processes ($P(k) = \text{const.}$ for small k) [3].

This is the behavior that one would like to detect in the data in order to confirm inflationary models. Up to now this search has been done through the analysis of the galaxy PS which has to go correspondingly as $P(k) \sim k$ at small k (large scales). No observational test of this behavior has been provided yet.

3 The gravitational many-body problem

As mentioned, the standard model of the formation of large scale structure of the universe is based on the gravitational growth of small initial density fluctuations in a homogeneous and isotropic medium (e.g., [1]). In the CDM model particles interact only gravitationally and they are cold, i.e. with very small initial velocity dispersion. This situation allows to model this system with a collision-less Boltzmann equation and, for sufficiently large scales, pressure-less fluid equations. Then it is possible to solve in a perturbative way, for small density fluctuations, these fluid equations (for a review see e.g. [1]). However this treatment is inapplicable in the strong non-linear regime. Then, the most widely used tool to study gravitational clustering in the various regimes is by means of N-body simulations (NBS) which are based on the computation of particle gravitational dynamics in an expanding universe.

One may consider an infinite periodic system, i.e. a finite system with periodic boundary condition. Despite the simplicity of the system, in which dynamics is Newtonian at all but the smallest scales, the analytic understanding of this crucial problem is limited to the regime of very small fluctuations where a linear analysis can be performed. As mentioned, the problem is Newtonian but the equation of motions are modified because of the expanding background. It is possible to consider some simplified cases where the expansion is not included and then study the differences introduced by space expansion.

An additional important point should be stressed: for numerical reasons, the cosmological density field must be discretized into "macro-particles" interacting gravitationally which are tens of order of magnitudes heavier than the (elementary) CDM particles due to computer limitations. This procedure introduces discreteness at a much larger scale than the discreteness inherent to the CDM particles. By discreteness we mean statistical and dynamical effects which are not described by the self-gravitating fluid approximation. The discreteness has different manifestations in the evolution of the system (see e.g. [6] and references therein). It is therefore necessary to consider the issue of the physical role of discrete fluctuations in the dynamics, which go beyond a description where particles play the role of collision-less fluid elements and the evolution can be understood in terms of a self-gravitating fluid.

In order to study the full gravitational many-body problem, we have considered a very simple initial particle distribution represented by a slightly perturbed simple cubic lattice with zero initial velocities [7]. A perfect cubic lattice is an unstable equilibrium configuration for gravitational dynamics, being the force on each particle equal to zero. A slightly perturbed lattice represents instead a situation where the force on each particle is small, and linearly proportional to the average root mean square displacement of any particle from its lattice position. When the system is evolved for long enough times it creates complex non-linear structures. While the full understanding of this clustering dynamics is not currently available, some steps have been done for what concerns the early times evolution of the system [7,8].

In this context an analogy with the dynamics of the Coulomb lattice (or Wigner crystal) helps to develop an analytical approach to the early time evolution of a gravitational infinite lattice of point mass particles, slightly perturbed about the equilibrium configuration. Apart a change in the sign of the force (in the Coulomb lattice case it is repulsive) the equation of motion is identical in the gravitational and Coulomb cases [7]. This allows us to quantitatively characterize and understand, in the linear approximation, the deviation of a finite number of particle system from the evolution of a self-gravitating fluid. This is relevant for the problem of cosmological structure formations, where the fluid approximation is usually used to model a system of large number of elementary dark matter particles while the simulations used to study numerically the problem employ a relative small number of particles [7,8].

This analogy is extremely useful to treat the particular problem of the evolution of a shuffled lattice at early times, i.e. before nearest particles collide. Then a study of the Poisson distribution [8] is very useful to treat the formation of the first power-law correlated structures. It is then surprising that the early-times power-law correlation function remains invariant during the subsequent time evolution, when structures are formed by many particles. A study of the late times dynamics is currently under consideration.

4 Large scale structures of the universe

The third topic we will briefly discuss is represented by the statistical properties of the matter distribution observed today through the study of three-dimensional galaxy catalogs [9]. Galaxy correlation properties seem to be similar to those of a fractal object [10]. Several issues of data analysis and the relation to cosmological simulations of structure formation are discussed, stressing the subtle ways in which the hypothesized dark matter enter into play. This distribution represents the observational test for any theory of cosmological structure formation.

In the past twenty years observations have provided several three dimensional maps of galaxy distribution, from which there is a growing evidence of large scale structures. This important discovery has been possible thanks to the advent of large redshift surveys: angular galaxy catalogs, considered in the past, are in fact essentially smooth and structure-less. Figure 1 shows a slice of the Center for Astrophysics galaxy catalog (CfA2), which was completed in the early nineties [11], and a slice derived from the recent observations of the Sloan Digital Sky Survey (SDSS) project [9]. In the CfA2 catalog, which was one of the first maps surveying the local universe, it has been discovered the giant "Great Wall" a filament linking several groups and clusters of galaxies of extension of about 200 Mpc/h[1] and whose size is limited by the sample boundaries. Recently the SDSS project has reveled the

[1] The typical mean separation between nearest galaxies is of about 0.1 Mpc. By local universe one means scales in the range $[1, 100]$ Mpc/h, where space geometry is basically Euclidean and dynamics is Newtonian, i.e. effects of General Relativity are negligible. On larger scales instead, one has to consider that relativistic corrections start play a role for the determination of the space geometry and dynamics. The size of the universe,

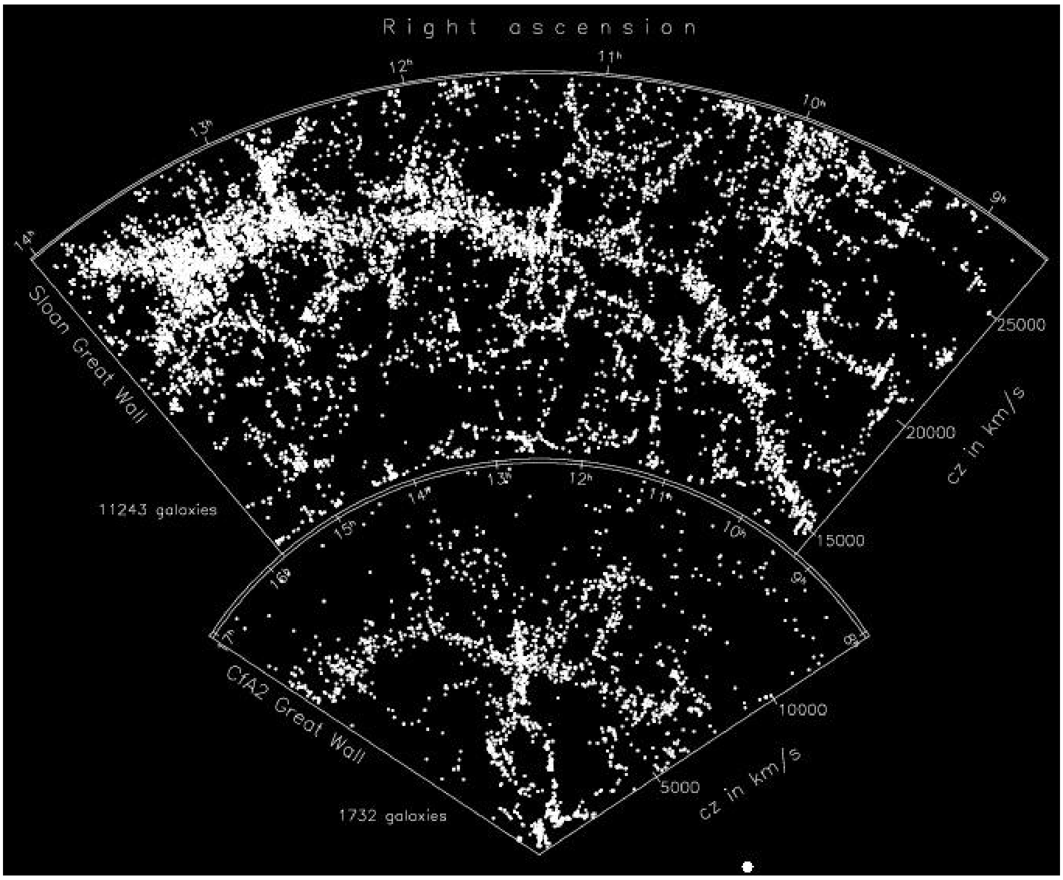

Fig. 1. Progress in redshift surveys: it is reported the "slice of the universe" from the CfA2 redshift survey [11] (lower part) and the new SDSS data [12] (upper part). This cone diagram represents the reconstruction of a thin slice observed from the Earth which is in the bottom. The CfA2 slice has an depth of 150 Mpc/h, while the SDSS slice has a depth of 300 Mpc/h. The "Great Wall" in the CfA2 slice and the new "Sloan Great Wall" in the SDSS slice are the dominant structures in these maps and they are clearly recognizable. For comparison we also show a small circle of size of 5 Mpc/h (bottom of the figure), the typical clustering length separating the regime of large and small fluctuations according to the standard analysis. (Elaboration from [2].)

existence of structures larger than the Great Wall, and in particular in Figure 1 one may notice the so-called "Sloan Great Wall" which is almost double longer than the Great Wall. Nowadays this is the most extended structure ever observed, covering about 400 Mpc/h, and whose size is again limited by the boundaries of the sample [12].

The search for the "maximum" size of galaxy structures and voids, beyond which the distribution becomes essentially smooth, is still an open problem. Instead the fact that galaxy structures are strongly irregular and form complex patterns has become a well-established fact. From the theoretical point of view the understating of the statistical characterization of these structures represents the key element to be considered by a physical theory dealing with their formation. The primary questions that such a situation rises are therefore: (i) which is the nature of galaxy structures and (ii) which is the maximum size of structures? A number of statistical concepts can be used to answer to these questions: in general one wants to characterize n-point correlation properties which are able to capture the main elements of points distributions [2].

Recently a team of the SDSS collaboration [13] has measured the conditional density $\langle n(r)\rangle_p$ (which gives the average density at distance r from an occupied point) as a function of scale in a sample of the SDSS survey which covers, to date, the largest volume of space ever considered for such an analysis with a very robust statistics and precise photometric calibration (Fig. 1). They found that: (i) there is clearly a "fractal regime" where $\langle n(r)\rangle_p \sim r^{D-3}$ with a dimension $D \approx 2$, which appears to terminate at somewhere between 20 and 30 Mpc/h – this behavior agrees very well with what we found at the scales we could probe properly (i.e. by making the full volume average) with the samples at our disposal a few years ago [19] and recently with the new 2dF sample [15] (see discussion

according to standard cosmological models is about 5000 Mpc/h, where 1 Mpc $\simeq 3 \times 10^{22}$ m; distances are given in units of h, a parameter which is in the range $[0.5, 0.75]$ reflecting the incertitude in the value of the Hubble constant ($H = 100$ h km s^{-1} Mpc^{-1}) used to convert redshift z into distances $d \approx c/Hz$ (where c is the velocity of light).

in [10]); (ii) the data show then a slow transition to homogeneity in the range $30 < r < 70$ Mpc/h, where a flattening of the conditional density seems to occur for scales larger than $\lambda_0 \approx 70$ Mpc/h, a scale comparable with the sample size precisely where its statistical validity becomes weaker. Note that often in the past, samples have shown finite size effects which produced this type of behavior, which was then eliminated by deeper samples (see e.g. [20]). For example such a high value of λ_0 implies that *all* previous determinations of the characteristic clsuetring length r_0 are biased by finite size effects [10]. In fact the estimated r_0 has grown of about a factor 3 from 5 Mpc/h to about 13 Mpc/h in the most recent data [18,10].

Whether the latest measurements will remain stable in future larger samples is a key issue to be determined, and this is directly related to the reality of the flattening at 70 Mpc/h: this will be clarified soon, as the volume surveyed by the SDSS will increase rapidly in the near future.

5 Conclusions

Statistical properties of primordial density fields show interesting analogies with systems in statistical physics, like the one-component plasma, whose main characteristic is the ordered, or super-homogeneous, nature. In the FRW models the super-homogeneous (or Harrison-Zeldovich) condition arises as a kind of consistency constraint: other, more inhomogeneous, stochastic fluctuations, like the uncorrelated Poisson case, will always break down in the FRW models in the past or future as the amplitude of perturbations in the gravitational potential may become arbitrarily large. We discussed that the observational detection of the super-homogeneous character of the matter density field, through the observation of galaxy distribution or of the CMBR anisotropies, is still lacking. On the other hand the main feature of galaxy two-point correlation function is represented by its power-law character in the strongly non-linear region. We stressed that a clear crossover to homogeneity is also not well established in the data, and thus the transition from the highly clustered phase to the highly uniform (super-homogeneous) one is the main observational tests for theories of the early universe. For galaxies this should be evidenced as a negative correlation function behaving as $-r^{-4}$ (corresponding to $P(k) \sim k$) at large scales. In this way one may have some constraints on the large number of free parameters which characterize cosmological dark matter, the main source for the seeds of structure formation in the universe according to standard models.

The theoretical understating of non-linear structure formation of a self-gravitating infinite particle distribution is a fascinating problem and many questions are still open. We discussed the fact that at early times, starting from an instable equilibrium configuration as a simple cubic lattice of point mass particles, it is possible to develop a stringent analogy with the Coulomb lattice dynamics. In this way it is possible to characterize and understand the deviation of a finite number of particle system from the evolution of a self-gravitating fluid. That is, it is possible to quantify the effects of discreteness and their role in the formation of non-linear structures. In this respect the main problem of cosmological simulations is that, because of the discretization used for numerical limitations, they could be affected by discrete effects, as a particle in the simulations has a mass which is many orders of magnitude larger than the elementary dark matter particles one would like to simulate. The understanding of the full time evolution, and of the creation of non-linear structures made by many particles is still the main open problem in this context.

We thank T. Baertschiger, A. Gabrielli, M. Joyce and B. Marcos for fruitful collaborations.

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